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REPORT R-1813

AN INTERIOR BALLISTIC ANALYSIS
OF A
HIGH-LOW PRESSURE GUN
USING NOZZLE-START, SHOT-START
AND
SECONDARY IGNITER MATERIAL

by

SIDNEY GOLDSTEIN
and
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**Small Caliber Engineering Directorate
FRANKFORD ARSENAL
Philadelphia, Pa. 19137**

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ABSTRACT

Interior ballistic equations suitable for computer analysis are developed to predict the performance of the 40mm grenade launcher M79 firing the multiple projectile cartridge XM576. These equations take into account the effects of the nozzle-start and shot-start pressures which are characteristic of this system. Also considered is the effect of secondary igniter material upon the ballistic performance. An isothermal model is used for the period during which the propellant is burning. Only propellant of constant burning surface area is considered in this study.

GLOSSARY OF SYMBOLS

A	Bore Area	in.²
B	Exponential Burning Rate Coefficient	in./sec-(Psi)ⁿ
B'	Linear Burning Rate Coefficient	in./sec-Psi
C	Propellant Weight	lbs
D	Propellant Web	in.
g	acceleration Due to Gravity	ft/sec²
n	Burning Rate Exponent	Dimensionless
N₁	Fraction of Charge Burnt Which Remains in High Pressure Chamber	Dimensionless
N₂	Fraction of Charge Burnt Which is Discharged Into Low Pressure Chamber	Dimensionless
P₁	Pressure in High Pressure Chamber	psi
P₂	Pressure in Low Pressure Chamber	psi
P_{ns}	Nozzle-Start Pressure	psi
P_{ss}	Shot-Start Pressure	psi
S	Total Throat Area of The Nozzles	in.²
t	Time	seconds
t_b	Time at Which Charge is Burnt	seconds
t_o	Time at Which Nozzles First Start to Discharge	seconds
U₁	High Pressure Chamber Volume	in.³
U₂	Low Pressure Chamber Volume	in.³

GLOSSARY OF SYMBOLS (Cont'd)

V	Velocity of Projectile	in. / sec
V_B	Velocity at All-Burnt	in. / sec
W	Projectile Weight	lbs
x	Travel	in.
x_B	Travel at All-Burnt	in.
γ	Ratio of Specific Heats	Dimensionless
ζ	Heat Loss Coefficient	Dimensionless
δ	Propellant Density	lbs/in.³
η	Co-Volume of Propellant	in.³/lb
λ	Effective Mean Impetus	ft-lbs/lb
Δ₀	Loading Density	gms/cm³
ψ	$\gamma^{1/2} \left[2/(\gamma + 1) \right]^{(\gamma + 1)/2(\gamma - 1)}$	Dimensionless
Ψ	$(\psi/\sqrt{\lambda/g}) (SD/2 B'C)$	Dimensionless
ψ	Fraction of the Charge Burnt	Dimensionless
ψ₀	Fraction of the Charge Which is Burnt at the Time the Nozzles First Start to Discharge	Dimensionless
ψ_s	Fraction of the Charge Which is Burnt at the Time the Projectile First Starts to Move	Dimensionless
ψ_i	Effective Fractional Charge Increase from Burning Igniter Material	Dimensionless

GLOSSARY OF SYMBOLS (Cont'd)

θ'	$U_1 / 6 (\gamma - 1) S \sqrt{\lambda g}$	seconds
τ	Nondimensional Time Factor Used to Describe the Period After All-Burnt	Dimensionless

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INTRODUCTION

The interior ballistic system for the 40mm grenade launcher firing a multishot antipersonnel round is unique. It differs from that used in conventional small arms in that it is a high-low ballistic system operating with shot-start and nozzle-start mechanisms. "Nozzle-start" means that the gas does not discharge from the high pressure chamber of the system until a blow-out disc which seals the nozzles ruptures. This does not occur until a certain pressure (nozzle-start pressure, P_{ns}) is reached and a certain fraction (q_o) of the charge is burnt. Also, the "shot-start" condition is not reached until a certain pressure ("shot-start" pressure P_{ss}) is obtained in a low pressure chamber. Likewise, this does not occur until a certain fraction (q_s) of the charge is burnt. A shot-start pressure is obtained by simply crimping the projectile in the cartridge case.

A computer study has been conducted to determine how changes in nozzle-start pressure and shot-start pressure together with changes in nozzle throat area and charge weight affect the performance of the system. This study will appear in a separate Frankford Arsenal report.^{1*}

The interior ballistics for a high-low gun have already been investigated by Corner² and Kapur³. However, these analyses are not adequate for the system described in this report since they do not take into account the nozzle-start and shot-start pressures and also the igniter material. Since it was felt that these factors would significantly affect peak pressures and velocity they were, therefore, taken into account in this study. Also, references 2 and 3 both use a term which represents the linear burning rate constant for web regression.** This quantity is most difficult to obtain without testing the system experimentally and measuring the maximum pressure. In this report an iterative method is presented for determining β or B' from a knowledge of the gun and propellant parameters.

*See references.

**In general there are two burning rate constants used in ballistics. β is used to determine the rate at which the web burning from both sides is diminished, and B' as used in this report to show the rate at which the web burning from one side is diminished (i.e. $B' = \beta/2$).

Numerical integration of the nonlinear differential equations involved in the interior ballistics of this system was performed on a digital computer using a Runge-Kutta method.¹

ASSUMPTIONS

1. The temperature of the gases during the burning period was assumed constant. This is equivalent to saying that the impetus (λ) is constant during the entire burning period. This applies to the gas in both the high and low chambers and in the bore of the gun. For numerical analysis¹ a value of λ equal to 0.90 of the true propellant impetus was used. This assumption was necessary to minimize the difficulties encountered in obtaining a solution.

2. The surface area of the propellant is assumed to remain constant during the burning period. This is equivalent to assuming the propellant form factor^{2, 3} is zero and that no unburnt propellant is ejected to the low pressure chamber.

3. The ratio of the pressure in the low pressure chamber (P_2) to that in the high pressure chamber P_1 is assumed to be always less than $[2/(\gamma + 1)]^{\gamma/(\gamma - 1)}$ (i.e. $P_2/P_1 < [2/(\gamma + 1)]^{\gamma/(\gamma - 1)}$). Thus the flow from the high pressure chamber is assumed to be sonic.

4. Covolume effects on the flow between the high and low pressure chambers are neglected.

5. The total heat loss up to any time t after shot-start is proportional to the kinetic energy of the projectile at that time. ζ is used to denote the constant of proportionality.

6. There exists a uniform gas temperature and pressure throughout the volume behind the projectile.

7. Flow in the high pressure chamber is assumed to be quasi-steady so that no pressure waves or gradients are introduced into the system.

8. Propellant ignition is assumed isochoric.

THEORY

Derivation of Equations

Internal Ballistics of Gun Before All-Burnt

1. Equation of State For High Pressure Chamber

$$P_1 = \frac{12 C \lambda N_1}{V_1 - C(1 - \xi)/\delta - CN_1 \eta} \quad (1)$$

2. Equation of State For Low Pressure Chamber

$$P_2 = \frac{12 C \lambda N_2}{V_2 + Ax - CN_2 \eta} \quad (2)$$

3. Equation of Web Regression

$$D \frac{d\psi}{dt} = 2 B' P_1 \quad (3)$$

4. Equation of Continuity For High Pressure Chamber

$$\frac{dN_1}{dt} = \frac{d\psi}{dt} - \frac{\psi S P_1}{C \sqrt{\lambda/g}} \quad (4a)$$

From the equation of web regression (eq 3) substitute for P_1 in the above equation (4a)

$$\frac{dN_1}{dt} = \frac{d\psi}{dt} - \frac{\psi S D}{2 B' \sqrt{\lambda/g} C} \frac{d\psi}{dt} \quad (4b)$$

the flow factor ψ is defined as

$$\psi = \frac{\psi SD}{2 B' \sqrt{\lambda/g} C} \quad (4c)$$

Substituting for ψ in (4b):

$$\frac{dN_1}{dt} = \frac{d\Phi}{dt} (1 - \psi) \quad (4d)$$

Integrating using the boundary $\Phi = \Phi_0$ when $N_1 = N_0 = \Phi_0$

$$N_1 = \Phi \left[1 - \psi (\Phi - \Phi_0) / \Phi \right] \quad (4e)$$

It should be noted at this point that Φ_0 represents the fraction of the charge which is burnt at the time when the nozzle first starts to discharge. In "Appendix A" it is shown that it is possible to take into account the effect of the igniter material (igniter weight - C_i , impetus - λ_i) upon the performance of the gun. To do this it is necessary that the propellant weight (C_p) be suitably adjusted and a Φ_i determined based upon the assumption that the total igniter charge is burnt before any of the propellant starts burning.

5. Equation of Continuity For Low Pressure Chamber

$$N_2 = \Phi - N_1 = \psi (\Phi - \Phi_0) \quad (5)$$

6. Pressure-Time Curve in High Pressure Chamber Before All-Burnt

a. Derivation of Pressure-Time Equation for High Pressure Chamber From Ignition to Nozzle-Start, ($\Phi = \Phi_0$)

Assuming that $\psi = 0$ for the period of ignition to nozzle start the pressure in the high pressure chamber is given by

$$P_1 = \frac{12 C \lambda \Phi}{V_1 - C(1 - \Phi)/\delta - C\eta\Phi} = \frac{D}{2 B'} \frac{d\Phi}{dt} \quad (6a)$$

or

$$\frac{d\Phi}{dt} = \frac{24 B' C \lambda \Phi}{D \left[V_1 - C/\delta + \Phi(C/\delta - C\eta) \right]} \quad (6b)$$

or

$$\frac{d\Phi}{dt} = \frac{\epsilon \Phi}{\Lambda + \Gamma \Phi} \quad (6c)$$

where

$$\epsilon = 24 B' C \lambda / D$$

$$\Gamma = C/\delta - C\eta \quad (6d)$$

$$\Lambda = V_1 - C/\delta$$

Integrating (6c) with the initial condition $\Phi = \Phi_i$ when $t = 0$ the following is obtained

$$\Gamma/\epsilon (\Phi - \Phi_i) + \Lambda/\epsilon \ln(\Phi/\Phi_i) = t \quad (6e)$$

$$0 \leq t \leq t_0$$

where Φ_i is determined by the amount of igniter material burned initially (see Appendix A). Having thus determined Φ as a function of time it is then possible using (6a) to determine P_1 as a function of time for this period.

b. Derivation of Pressure-Time Equation for High Pressure Chamber From Nozzle-Start Until All-Burnt

Substituting equation (4e) for N_1 in equation (1) the following equation is obtained:

$$P_1 = \frac{12 C \lambda \Phi \left[1 - \Psi (\Phi - \Phi_0) / \Phi \right]}{U_1 - C(1 - \Phi) / \delta - C \eta \Phi \left[1 - \Psi (\Phi - \Phi_0) / \Phi \right]} \quad (6f)$$

substituting for P_1 in equation (3)

$$P_1 = \frac{D}{2B'} \frac{d\Phi}{dt} = \frac{12 C \lambda \Phi \left[1 - \Psi (\Phi - \Phi_0) / \Phi \right]}{U_1 - C(1 - \Phi) / \delta - C \eta \Phi \left[1 - \Psi (\Phi - \Phi_0) / \Phi \right]} \quad (6g)$$

Or

$$\frac{d\Phi}{dt} = \frac{\xi' + \epsilon' \Phi}{\Lambda' + \Gamma' \Phi} \quad (6h)$$

where

$$\xi' = 24 B' C \lambda \Psi_0 \Psi / D$$

$$\epsilon' = 24 B' C \lambda (1 - \Psi) / D$$

$$\Lambda' = U_1 - C / \delta - C \eta \Psi \Phi_0$$

$$\Gamma' = C / \delta - C \eta + C \eta \Psi$$

Integrating (6h) using the initial conditions $\Phi = \Phi_0$ at $t = t_0$ the following is obtained:

$$\Gamma' / \epsilon' (\Phi - \Phi_0) + K / \epsilon' \ln(\xi' + \epsilon' \Phi / \xi' + \epsilon' \Phi_0) = t - t_0 \quad (6i)$$

where

$$t_0 \leq t \leq t_b$$

where

$$K = (\epsilon' \Lambda' - \xi' \Gamma') / \epsilon'$$

and t_b is the time to all-burnt ($\phi = 1$). Having thus determined ϕ as a function of time it is then possible using (6f) to determine P_1 as a function of time for this period of time.

7. Equation of Motion

$$\frac{W}{12g} V \frac{dV}{dx} = A P_2 \quad (7a)$$

now

$$D \frac{d\phi}{dt} = D \left(\frac{dx}{dt} \right) \left(\frac{d\phi}{dx} \right) = DV \frac{d\phi}{dx} = 2 B' P_1$$

or

$$V = \frac{2 B' P_1}{D} \left(\frac{dx}{d\phi} \right) \quad (7b)$$

$$\frac{dV}{d\phi} = \frac{2 B'}{D} \frac{d}{d\phi} \left[P_1 \left(\frac{dx}{d\phi} \right) \right] \quad (7c)$$

$$\frac{dV}{dx} = \frac{dV}{d\phi} \frac{d\phi}{dx} = \frac{dV}{d\phi} \left(\frac{2 B' P_1}{DV} \right) \quad (7d)$$

substituting (7c) into (7d)

$$\begin{aligned} \frac{dV}{dx} &= \frac{2 B' P_1}{DV} \left[\frac{2 B'}{D} \frac{d}{d\phi} \left(P_1 \frac{dx}{d\phi} \right) \right] \\ &= \frac{4 B'^2 P_1}{D^2 V} \frac{d}{d\phi} \left[P_1 \frac{dx}{d\phi} \right] \end{aligned} \quad (7e)$$

substituting (7e) into (7a)

$$\frac{d}{d\phi} \left[P_1 \frac{dx}{d\phi} \right] = \frac{3 A D^2 g}{B^2 W} \left(\frac{P_2}{P_1} \right)$$

or

$$\frac{dP_1}{d\Phi} \left(\frac{dx}{d\Phi} \right) + P_1 \frac{d^2x}{d\Phi^2} = \frac{3AD^2g}{B^{12}W} \left(\frac{P_2}{P_1} \right) \quad (7f)$$

since

$$P_1 = P_1(N_1, \Phi) \text{ and } N_1 = N_1(\Phi)$$

now

$$\frac{dP_1}{d\Phi} = \left(\frac{\partial P_1}{\partial N_1} \right)_{\Phi} \frac{dN_1}{d\Phi} + \left(\frac{\partial P_1}{\partial \Phi} \right)_{N_1}$$

from (4e)

$$N_1 = \Phi \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right]$$

$$dN_1/d\Phi = 1 - \Psi \quad (7g)$$

using (1)

$$\left(\frac{\partial P_1}{\partial N_1} \right)_{\Phi} = \frac{12 C \lambda \left[U_1 - C/\delta (1 - \Phi) \right]}{\left[U_1 - C/\delta (1 - \Phi) - C N_1 \eta \right]^2} \quad (7h)$$

likewise

$$\left(\frac{\partial P_1}{\partial \Phi} \right)_{N_1} = - \frac{12 C \lambda C/\delta \left\{ \Phi \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right] \right\}}{\left[U_1 - C/\delta (1 - \Phi) - C N_1 \eta \right]^2} \quad (7i)$$

$$\therefore \frac{dP_1}{d\Phi} = \frac{12 C \lambda \left\{ \left[U_1 - C/\delta (1 - \Phi) \right] \left[1 - \Psi \right] - C\Phi/\delta \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right] \right\}}{\left\{ U_1 - C/\delta (1 - \Phi) - C\eta\Phi \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right] \right\}^2} \quad (7j)$$

thus (7f) reduces to

$$\frac{dx}{d\Phi} \left\{ \frac{12 C \lambda \left\{ \left[U_1 - C/\ell - \Phi \right] (1 - \Psi) - C\Phi/\delta \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right] \right\}}{\left(U_1 - C/\delta (1 - \Psi) - C\eta\Phi \left[1 - \Psi(\Phi - \Phi_0)/\Phi \right] \right)^2} \right\} + P_1 \left(\frac{d^2 x}{d\Phi^2} \right) = \frac{3 A D^2 g}{B^{12} W_2} \left(\frac{P_2}{P_1} \right) \quad (7k)$$

where equations (1) and (2) are now substituted for P_1 and P_2 .

This yields a nonlinear differential equation which is integrated numerically using the method of Runge-Kutta.¹ The boundary conditions are $dx/d\Phi = 0$, $x = 0$ when $\Phi = \Phi_s$. From this equation it is then possible to determine the travel and velocity as functions of Φ and thence using equation (6i) as functions of time.

8. Equation for Maximum Pressure in High Pressure Chamber

The primary difficulty in determining peak pressure from equation (7j) is that Ψ is a function of B' which is in itself a function of peak pressure.

$$\Psi = \frac{\psi S D}{\sqrt{\lambda/g} \ 2B'C} \quad (8a)$$

Φ_s is the fraction of the charge which is burnt at the time the projectile first starts to move. At this time the pressure is equal to the shot-start pressure (P_{ss}). Φ_s may be calculated from equation (2) by substituting $P_{ss} = P_2$ and $\Phi_s = \Phi$ and letting $x = 0$. Thus Φ_s is given by

$$\Phi_s = \Phi_0 + \frac{P_{ss} U_2}{12 C \lambda \Psi + P_{ss} C \eta \Psi}$$

B' is usually determined by using the value of peak pressure obtained from experimental data, or the peak pressure is estimated and then B' is obtained.

One way of determining B' from the peak pressure data obtained experimentally is to select that value of B' such that the area under the assumed linear burning rate ($r' = B'P$) curve equals that under the exponential burning rate ($r = BP^n$) curve. The limits are taken from $P = 0$ to $P = P_{\max}$.

Thus

$$\int_0^{P_{\max}} B'P dP = \int_0^{P_{\max}} BP^n dP$$

This gives

$$B' = \frac{2B}{(n+1) P_{\max}^{(1-n)}} \quad (8b)$$

Values of B and n can usually be found in literature or obtained from burning rate studies for different propellants.

From equation (7j) it may be seen that P_1 increases monotonically with Φ (i.e., $dP_1/d\Phi \geq 0$), provided that the following condition is met:

$$U_1 (1 - \Psi) - C/\delta [1 - \Psi(1 - \Phi_0)] \geq 0$$

With this condition P_1 will be maximum when Φ is maximum. This will occur at all-burnt ($\Phi = 1$). Since loading density in gm/cm^3 is:

$$\Delta_0 = 27.7 C/U_1$$

the condition for maximum pressure to occur at all-burnt is thus

$$C/U \leq \delta (1 - \Psi) / [1 - \Psi(1 - \Phi_0)]$$

or

$$L_0 = 27.7 C/U \leq 27.7 \xi (1 - \Psi) / 1 - \Psi(1 - \Phi_0)$$

Some values of Ψ and Φ_0 which appear feasible are:

$$0.40 < \Psi < 0.85$$

$$0.02 < \Phi_0 < 0.20$$

Taking the case $\Psi = 0.40$ and $\Phi_0 = 0.20$ the loading density Δ_0 would then have to be less than about 0.97 gms/cm^3 for peak pressure to occur at all-burnt. Thus our original assumption that peak pressure occurs at all-burnt appears to be a reasonable one for most practical high-low guns using propellant with a constant burning surface.

The equation for the peak pressure is then given by (1) with $\Phi = 1$.

$$P_{1m} = \frac{12 C \lambda [1 - \Psi(1 - \Phi_0)]}{U_1 - C \eta [1 - \Psi(1 - \Phi_0)]} \quad (8c)$$

As mentioned previously Ψ which is dependent upon B' is also a function of P_{1m} . Thus an iterative technique was used to determine P_{1m} . First a value of Ψ is assumed. This value is then used in equation (8c) to determine P_{1m} . The value of B' is then determined from equation (8b) and thence using this value of B' a new value of Ψ calculated from equation (8a). This is compared with our original estimate of Ψ and the procedure continued until both agree. The peak pressure is then also established from equation (8c).

Internal Ballistics of Gun After All-Burnt

For the period after all-burnt the isothermal model will be neglected and the gases in the high pressure chamber assumed to undergo an adiabatic expansion. The flow of the gas likewise will be assumed to be sonic during this period.

9. Pressure In The High Pressure Chamber After All-Burnt

Following Corner's approach it may be shown^{1, 2} that after all-burnt the equation for the pressure in the high pressure chamber as a function of time is given by:

$$P_1 = \frac{12 C \lambda [1 - \Psi(1 - \Phi_0)]}{U_1 - \eta C [1 - \Psi(1 - \Phi_0)]} \left(1 + (t - t_b)/\theta\right)^{-2\gamma/\gamma-1} \quad (9a)$$

$N_1 = N_1'$ after all-burnt and is given by

$$N_1' = [1 - \Psi(1 - \Phi_0)] \left(1 + (t - t_b)/\theta\right)^{-2/\gamma-1} \quad (9b)$$

and

$$\theta_1 = \frac{2 U_1}{12 (\gamma - 1) \psi S (\lambda g)^{1/2}} \quad (9c)$$

10. To Determine the Muzzle Velocity After All-Burnt

The pressure in the low pressure chamber after all-burnt is given by:

$$P_2 = \frac{12 C \lambda N_2' \tau_2}{U_2 + A x - C \eta N_2'}$$

Since

$$N_2' = (1 - N_1')$$

we have

$$P_2 = \frac{12 C \lambda (1 - N_1') \tau_2}{U_2 + A x - C \eta (1 - N_1')} \quad (10a)$$

The equation of motion has already been given as:

$$\frac{W_2}{12g} V \frac{dV}{dx} = \frac{W_2}{12g} \frac{d^2x}{dt^2} = A P_2 \quad (7a)$$

The rate of change in internal energy of the gases in both chambers is given by:

$$\dot{E}_1 = \frac{12 C \lambda}{\gamma - 1} \frac{d}{dt} \left[N_1' \tau_1 + (1 - N_1') \tau_2 \right] \quad (10b)$$

where τ_1 is the ratio of the gas temperature in the high pressure chamber after all-burnt to the temperature which it had during burning, and likewise τ_2 is the ratio of the gas temperature in the low pressure chamber after all-burnt to the temperature which it had during burning.

The rate at which work is done on the projectile is equal to the rate of change of kinetic energy:

$$\dot{E}_2 = \frac{1}{24} \frac{W}{g} \frac{d}{dt} \left[(dx/dt)^2 \right] \quad (10c)$$

If it is assumed that heat loss for any period of time after shot-start is proportional to kinetic energy of the projectile at that instant then the rate of loss of energy by heat transfer can be written as:

$$\dot{E}_3 = \frac{\zeta W}{24g} \frac{d}{dt} \left[(dx/dt)^2 \right] \quad (10d)$$

where ζ is the proportionality constant.

The sum of (10b), (10c) and (10d) must equal zero since the total energy of the system is constant.

Let $(\bar{\gamma} - 1) = (\gamma - 1)(1 + \zeta)$ in the above sum. Integrating this sum and making use of the following initial conditions $N_1' = N_{1b}$, $\tau_1 = \tau_{1b} = 1$, $\tau_2 = \tau_{2b} = 1$ (the subscript b indicates the condition at all-burnt) when $(dx/dt) = (dx/dt)_b$ we obtain:

$$\begin{aligned} \frac{12 C \lambda}{\bar{\gamma} - 1} \left[N_1' \tau_1 - N_{1b} + (1 - N_1') \tau_2 - (1 - N_{1b}') \right] = \\ - \frac{1}{24} \frac{W}{g} \left[(dx/dt)^2 - (dx/dt)_b^2 \right] \end{aligned} \quad (10e)$$

Using equation (7a), the equation of state for the low pressure chamber after all-burnt (equation (2)) may be rewritten as:

$$\begin{aligned} 12 C \lambda (1 - N_1') \tau_2 &= P_2 \left[U_2 + Ax - C(1 - N_1') \eta \right] \\ &= \frac{W}{12 g A} \frac{d^2 x}{dt^2} \left[U_2 + Ax - C(1 - N_1') \eta \right] \end{aligned} \quad (10f)$$

Using equation (9b), the equation of state for the high pressure chamber after all-burnt (equation (1)) may be rewritten as:

$$\begin{aligned} 12 C \lambda N_1' \tau_1 &= P_1 \left[U_1 - C N_1' \eta \right] \\ &= P_{1b} \left[U_1 - C N_1' \eta \right] \left[1 + (t - t_b)/\theta \right]^{-2\gamma/\gamma-1} \end{aligned} \quad (10g)$$

substituting 10f and 10g into 10e

$$\begin{aligned} P_{1b} \left[U_1 - C N_1' \eta \right] \left[1 + (t - t_b)/\theta \right]^{-2\gamma/\gamma-1} + \frac{W}{12 g A} \frac{d^2 x}{dt^2} \left[U_2 \right. \\ \left. + Ax - C(1 - N_1') \eta \right] - 12 C \lambda = - \frac{(\bar{\gamma} - 1) W}{24 g} \left[(dx/dt)^2 \right. \\ \left. - (dx/dt)_b^2 \right] \end{aligned}$$

or multiplying through by A and rearranging terms:

$$\begin{aligned} \frac{W}{12g} (d^2x/dt^2) [U_2 + Ax - C\eta(1 - N'_1)] + (\bar{\gamma} - 1)AW/24g (dx/dt)^2 \\ = (\bar{\gamma} - 1)WA/24g (dx/dt)_b^2 + 12AC\lambda - P_{1b}[U_1 \\ - CN'_1\eta] [1 + (t - t_b)/\theta]^{-2\gamma/\gamma-1} \end{aligned} \quad (10h)$$

where

$$N'_1 = N_{1B} [1 + (t - t_B)/\theta]^{-2/\gamma-1} = [1 - \psi(1 - \theta_0)] [1 + (t - t_b)/\theta]^{-2/\gamma-1}$$

and where

$$\theta = \frac{2U_1}{12(\gamma - 1)\psi S \sqrt{\lambda g}}$$

Equation (10h) is a nonlinear differential equation which is integrated numerically¹ using the method of Runge-Kutta. The boundary conditions are $x = x_B$ and $dx/dt = (dx/dt)_b$ when $t = t_b$.

CONCLUSIONS AND RECOMMENDATIONS

1. The assumptions made and the equations developed in this report appear reasonable for predicting the performance of a high-low pressure gun using nozzle-start, shot-start and secondary igniter material.

2. Future studies should be conducted to determine how the equations developed in this report could be modified to further describe high-low pressure guns where factors such as propellant loss between the chambers, progressiveness and degressiveness of the propellant and changes of gas temperature during burning are important.

APPENDIX A

To determine the effect of igniter material on the Interior Ballistic Performance.

C_P = Wt. of propellant (lbs)

C_i = Wt. of igniter (lbs)

C_2 = Effective charge wt of igniter (lbs)

λ = Impetus of propellant (ft-lbs/lb)

λ_i = Impetus of igniter (ft-lbs/lb)

C = Total effective charge wt. (lbs)

η_i = Covolume of igniter material (in.³/lb)

Assume that the total igniter charge is burnt at the time the propellant first starts to burn. Let Φ_i be the fraction of the total effective charge (C) burnt at this time.

$$\Phi_i = C_2 / (C_P + C_2) \quad (A-1)$$

Let P_i be the pressure generated by the igniter.

$$P_i = 12 C_i \lambda_i / U_1 - C_P / \delta - \eta_i C_i \quad (A-2)$$

Assume this igniter pressure to be generated by an initial charge burnt with an impetus (λ) equal to that of the propellant alone:

$$P_i = \frac{12 (C_P + C_2) \Phi_i \lambda}{U_1 - \frac{(C_P + C_2) (1 - \Phi_i)}{\delta} - (C_P + C_2) \eta_i \Phi_i} \quad (A-3)$$

Substituting for Φ_1 from equation (A-1)

$$P_1 = \frac{12 C_2 \lambda}{U_1 - C_P/\delta - C_2 \eta} \quad (A-4)$$

Solving for C_2

$$C_2 = \frac{U_1 - C_P/\delta}{12\lambda/P_1 + \eta} \quad (A-5)$$

Thus using (A-2) and (A-5) we can then determine

$$\Phi_1 = C_2 / (C_2 + C_P)$$

and the total effective charge

$$C = C_2 + C_P$$

Likewise, the total effective web is

$$D = \frac{D_P}{1 - \Phi_1}$$

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